Worksheet 7

1. Define a relation $\sim$ on $\mathbb{Z}$ by $a \sim b$ if and only if $5a \equiv 2b \ mod \ 3$

   Prove that $\sim$ is an equivalence relation

   (Note: $x \equiv y \ mod \ z$ simply means $x - y = kz$ for some $k \in \mathbb{Z}$)

2. Define a relation $\sim$ on $\mathbb{R} \times \mathbb{R}$ by $(a, b) \sim (c, d)$ if and only if $3(c - a) = d - b$

   Prove that $\sim$ is an equivalence relation

3. Define a relation $\sim$ on $\mathbb{R} \times \mathbb{R}$ by $(x, y) \sim (z, w)$ if and only if $x^2 + y^2 = z^2 + w^2$

   We proved $\sim$ is an equivalence relation in the last worksheet

   a. Describe the equivalence class $C_{(1,1)}$

   b. The above equivalence relation gives us a partition of $\mathbb{R} \times \mathbb{R}$, what is the index set of this partition?

   c. Can you come up with some other partitions of $\mathbb{R} \times \mathbb{R}$?

   d. Give the index set of the partition that you just found

e. (Challenge) Find the corresponding equivalence relation according to the partition you just found.

f. (Challenge) Prove the relation that you just found is an equivalence relation.

4. Let ~ be an equivalence relation on A and let $x, y \in A$.

   Prove $C_x \cap C_y \neq \emptyset$ if and only if $C_x = C_y$

5. For each $\alpha \in \mathbb{R}$, $A_\alpha = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = \alpha - x^2\}$

   Prove that $\mathcal{A} = \{A_\alpha \mid \alpha \in \mathbb{R}\}$ is a partition of $\mathbb{R} \times \mathbb{R}$

6. Prove that $\subseteq$ defines a partial ordering in on $\mathcal{P}(X)$